Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Further Mathematics

Advanced Subsidiary
Paper 1: Core Pure Mathematics

Sample Assessment Material for first teaching September 2017

Time: 1 hour 40 minutes

Paper Reference **8FM0/01**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. $$f(z) = z^3 + pz^2 + qz - 15$$

where p and q are real constants.

Given that the equation $f(z) = 0$ has roots

$$\alpha, \ \frac{5}{\alpha} \text{ and } \left(\alpha + \frac{5}{\alpha} - 1\right)$$

(a) solve completely the equation $f(z) = 0$

(5)

(b) Hence find the value of p.

(2)

1.a) $f(z) = z^3 + pz^2 + qz - 15 = 0$

roots : $\alpha, \ \frac{5}{\alpha}, \ \alpha + \frac{5}{\alpha} - 1$

$ax^3 + bx^2 + cx + d = 0$

$\alpha + \beta + \gamma = \frac{-b}{a}$ $\quad \alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$ $\quad \alpha\beta\gamma = \frac{-d}{a}$

$-\frac{d}{a} = -\frac{(-15)}{1} = 15 = $ product of all 3 roots

$\cancel{\alpha}\left(\frac{5}{\cancel{\alpha}}\right)\left(\alpha + \frac{5}{\alpha} - 1\right) = 15$

$5\alpha + \frac{25}{\alpha} - 5 = 15$ ← multiply by α

$5\alpha^2 - 20\alpha + 25 = 0$

$\alpha^2 - 4\alpha + 5 = 0$

$\alpha = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)}}{2}$

$= 2 \pm i$

QUADRATIC FORMULA:

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Question 1 continued

roots : α , $\frac{5}{\alpha}$, $\alpha + \frac{5}{\alpha} - 1$

$\therefore$ $2+i$, $2-i$, 3

b) $-\frac{b}{a} = -\frac{p}{1} =$ sum of roots

$-p = (2+i) + (2-i) + 3$

$-p = 7$

$p = -7$

(Total for Question 1 is 7 marks)

2. The plane Π passes through the point A and is perpendicular to the vector $\mathbf{n}$

Given that

$$\overrightarrow{OA} = \begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix} \quad \text{and} \quad \mathbf{n} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

where O is the origin,

(a) find a Cartesian equation of Π .

(2)

With respect to the fixed origin O, the line l is given by the equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix}$$

The line l intersects the plane Π at the point X.

(b) Show that the acute angle between the plane Π and the line l is 21.2° correct to one decimal place.

(4)

(c) Find the coordinates of the point X.

(4)

2.a) Equation of plane:

point on plane

$$\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$$

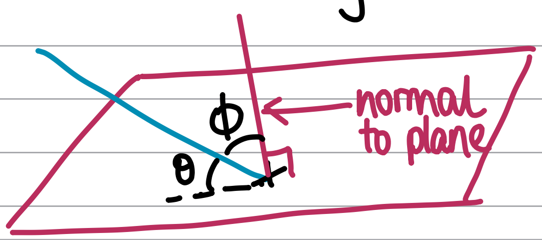
normal to plane

$$\Pi : \mathbf{r} \cdot \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = 5(3) - 3(-1) - 4(2) = 10$$

$$3x - y + 2z = 10$$

b)



Question 2 continued

$$\boxed{\cos\theta = \frac{a \cdot b}{|a||b|}}$$

direction vector of line, l

$$\cos\phi = \frac{d \cdot n}{|d||n|}$$ ← normal to plane

$$\cos\phi = \frac{\begin{pmatrix}-1\\-5\\3\end{pmatrix}\cdot\begin{pmatrix}3\\-1\\2\end{pmatrix}}{\left|\begin{pmatrix}-1\\-5\\3\end{pmatrix}\right|\left|\begin{pmatrix}3\\-1\\2\end{pmatrix}\right|} = \frac{-(3)-5(-1)+3(2)}{\sqrt{35}\sqrt{14}} = \frac{8}{\sqrt{35}\sqrt{14}}$$

$$\phi = 68.8°$$

$$\theta = 90° - 68.8° = 21.2°$$

c) The line, l, intersects the plane, Π at X

$$r = \begin{pmatrix}7\\3\\-2\end{pmatrix} + \lambda\begin{pmatrix}-1\\-5\\3\end{pmatrix} \qquad \Pi: 3x - y + 2z = 10$$

$$3(7-\lambda) - (3-5\lambda) + 2(-2+3\lambda) = 10$$

$$14 + 8\lambda = 10$$

$$\lambda = -\frac{1}{2}$$

$$x = \begin{pmatrix}7\\3\\-2\end{pmatrix} - \frac{1}{2}\begin{pmatrix}-1\\-5\\3\end{pmatrix} = \begin{pmatrix}7.5\\5.5\\-3.5\end{pmatrix} \qquad X = (7.5, 5.5, -3.5)$$

(Total for Question 2 is 10 marks)

3. Tyler invested a total of £5000 across three different accounts; a savings account, a property bond account and a share dealing account.

Tyler invested £400 more in the property bond account than in the savings account.

After one year

- the savings account had increased in value by 1.5%
- the property bond account had increased in value by 3.5%
- the share dealing account had **decreased** in value by 2.5%

- the total value across Tyler's three accounts had increased by £79

Form and solve a matrix equation to find out how much money was invested by Tyler in each account.

(7)

3 savings → £ x
property → £ y
share → £ z

write 3 equations using all known info.

$$x + y + z = 5000$$

$$y = x + 400$$

$$1.015x + 1.035y + 0.975z = 5079 \leftarrow \text{increased by £79}$$

$$\left.\begin{aligned} x + y + z &= 5000 \\ x - y &= -400 \\ 1.015x + 1.035y + 0.975z &= 5079 \end{aligned}\right\} \text{formed simultaneous equations}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1.015 & 1.035 & 0.975 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ -400 \\ 5079 \end{bmatrix}$$

$$MX = B$$
$$\cancel{M^{-1}}\,\cancel{M}X = M^{-1}B$$
$$X = M^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1.015 & 1.035 & 0.975 \end{bmatrix}^{-1} \begin{bmatrix} 5000 \\ -400 \\ 5079 \end{bmatrix} = \begin{bmatrix} 1800 \\ 2200 \\ 1000 \end{bmatrix}$$

∴ £1800 in savings
£2200 in property
£1000 in share

4. The cubic equation

$$x^3 + 3x^2 - 8x + 6 = 0$$

has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(\alpha - 1)$, $(\beta - 1)$ and $(\gamma - 1)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where p, q and r are integers to be found.

(5)

4. METHOD 1 (substitute in equation):

$x^3 + 3x^2 - 8x + 6 = 0$

└ roots α, β, γ

equation with roots: $\alpha - 1, \beta - 1, \gamma - 1$

Let $w = x - 1$

$x = w + 1$

$(w+1)^3 + 3(w+1)^2 - 8(w+1) + 6 = 0$

$w^3 + 3w^2 + 3w + 1 + 3w^2 + 6w + 3 - 8w - 8 + 6 = 0$

$w^3 + 6w^2 + w + 2 = 0$

METHOD 2 (using sum/product rules for polynomials):

$ax^3 + bx^2 + cx + d = 0$

$\alpha + \beta + \gamma = \frac{-b}{a}$ $\quad \alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$ $\quad \alpha\beta\gamma = \frac{-d}{a}$

$x^3 + 3x^2 - 8x + 6 = 0$

$\alpha + \beta + \gamma = -\frac{3}{1} = -3$ $\quad \alpha\beta + \alpha\gamma + \beta\gamma = \frac{-8}{1} = -8$ $\quad \alpha\beta\gamma = -\frac{6}{1} = -6$

DO NOT WRITE IN THIS AREA

Question 4 continued

equation with roots $(\alpha-1),(\beta-1),(\gamma-1) \rightarrow w^3+pw^2+qw+r=0$

$-p = \text{sum} = \alpha-1+\beta-1+\gamma-1 = \alpha+\beta+\gamma-3$
$= -3 -3 = -6$

$p = 6$

$q = (\alpha-1)(\beta-1)+(\alpha-1)(\gamma-1)+(\beta-1)(\gamma-1)$

$= \alpha\beta-\alpha-\beta+1+\alpha\gamma-\alpha-\gamma+1+\beta\gamma-\beta-\gamma+1$

$= \alpha\beta+\alpha\gamma+\beta\gamma-2\alpha-2\beta-2\gamma+3$

$= \alpha\beta+\alpha\gamma+\beta\gamma-2(\alpha+\beta+\gamma)+3$

$= -8-2(-3)+3 = 1$

$q = 1$

$-r = (\alpha-1)(\beta-1)(\gamma-1)$

$= (\alpha\beta-\alpha-\beta+1)(\gamma-1)$

$= \alpha\beta\gamma-\alpha\gamma-\beta\gamma+\gamma-\alpha\beta+\alpha+\beta-1$

$= \alpha\beta\gamma-(\alpha\beta+\beta\gamma+\alpha\gamma)+\alpha+\beta+\gamma-1$

$= -6-(-8)+(-3)-1$

$= -2$ $\qquad r = 2$

$w^3+6w^2+w+2=0$

(Total for Question 4 is 5 marks)

5. $$\mathbf{M} = \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$$

(a) Show that **M** is non-singular.
(2)

The hexagon R is transformed to the hexagon S by the transformation represented by the matrix **M**.

Given that the area of hexagon R is 5 square units,

(b) find the area of hexagon S.
(1)

The matrix **M** represents an enlargement, with centre (0, 0) and scale factor k, where $k > 0$, followed by a rotation anti-clockwise through an angle θ about (0, 0).

(c) Find the value of k.
(2)

(d) Find the value of θ.
(2)

5.a) $M = \begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$ Non-singular means that the determinant $\neq 0$

$\boxed{\text{DETERMINANT OF } 2\times2 \text{ MATRIX} \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \det(A) = |A| = ad - bc}$

$\det M = 1(1) - (\sqrt{3})(-\sqrt{3})$

$= 1 + 3$

$= 4 \neq 0$

$\therefore \det M \neq 0 \therefore M$ is non-singular

b) Determinant of matrix transformation gives scale factor of transformation to area

$\det M = 4 \quad \therefore \text{Area}(S) = 4 \times \text{Area}(R) = 4 \times 5 = 20$

Question 5 continued

c) If area increases by scale factor 4

└ enlargement by scale factor $\sqrt{4} = 2$

$\therefore k = 2$

d) $M = \underbrace{\text{enlargement}}_{\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}}$ followed by $\underbrace{\text{rotation}}_{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}$

$\boxed{\text{Transformation } A \text{ followed by } B \text{ is represented by the matrix } BA}$

$\therefore M = \text{rotation}(\text{enlargement})$

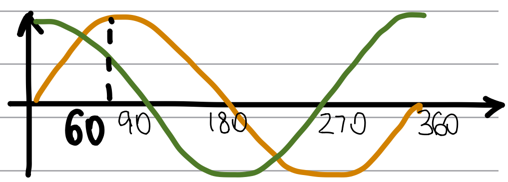
$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$$

$\cos\theta = \frac{1}{2}$ $\quad$ $\sin\theta = \frac{\sqrt{3}}{2}$



$\therefore \theta = 60°$

(Total for Question 5 is 7 marks)

6. (a) Prove by induction that for all positive integers n,

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

(6)

(b) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that for all positive integers n,

$$\sum_{r=1}^{n} r(r+6)(r-6) = \frac{1}{4}n(n+1)(n-8)(n+9)$$

(4)

(c) Hence find the value of n that satisfies

$$\sum_{r=1}^{n} r(r+6)(r-6) = 17\sum_{r=1}^{n} r^2$$

(5)

6.a) INDUCTION:
- Prove true for base case
- Assume true for $n=k$
- consider $n=k+1$ & replace by assumption
- Conclusion

Base case $n=1$:

LHS : $\sum_{r=1} r^2 = (1)^2 = 1$

RHS : $\frac{1}{6}(1)((1)+1)(2(1)+1)$

$= \frac{1}{6} \times 2 \times 3 = 1$

$\therefore$ statement true for $n=1$

Assume true for $n=k$

$$\sum_{r=1}^{k} r^2 = \frac{1}{6}k(k+1)(2k+1)$$

Consider $n=k+1$

$$\sum_{r=1}^{k+1} r^2 = (k+1)^2 + \sum_{r=1}^{k} r^2$$

$$= (k+1)^2 + \frac{k(k+1)(2k+1)}{6}$$

Question 6 continued

$$= \frac{6(k+1)^2 + k(k+1)(2k+1)}{6}$$

$$= \frac{(k+1)(6k+6+k(2k+1))}{6}$$

$$= \frac{(k+1)(2k^2+7k+6)}{6}$$

$$= \frac{(k+1)(2k+3)(k+2)}{6}$$

$$= \frac{1}{6}(k+1)((k+1)+1)(2(k+1)+1)$$

$\therefore$ hence true for $n=k+1$

Since statement is true for $n=1$, and we have proved if true for $n=k$, it is true for $n=k+1$, thus by mathematical induction, the result holds true for all positive integers

b) $\sum_{r=1}^{n} r(r+6)(r-6)$

$$= \sum_{r=1}^{n} r^3 - 36r$$

USING STANDARD SUMMATIONS

$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$ from formulae booklet

$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$ triangular no. formulae

$$= \frac{1}{4}n^2(n+1)^2 - 36\left(\frac{1}{2}n(n+1)\right)$$

Question 6 continued

$$= \frac{1}{4}\left(n^2(n+1)^2 - 72n(n+1)\right)$$

$$= \frac{1}{4}n(n+1)\left(n^2+n-72\right)$$

$$= \frac{1}{4}n(n+1)(n-8)(n+9)$$

c) $$\sum_{r=1}^{n} r(r+6)(r-6) = 17\sum_{r=1}^{n} r^2$$

$$\frac{1}{4}\cancel{n(n+1)}(n-8)(n+9) = 17\left(\frac{1}{6}\cancel{n(n+1)}(2n+1)\right)$$

$$\frac{1}{4}(n-8)(n+9) = \frac{17}{6}(2n+1)$$ (x 12 both sides)

$$3(n^2+n-72) = 34(2n+1)$$

$$3n^2+3n-216 = 68n+34$$

$$3n^2-65n-250 = 0$$

$$(n-25)(3n+10) = 0$$

$\therefore n = 25 \cup \cancel{-\frac{10}{3}}$ however n must be a positive integer

$\therefore n = 25$

(Total for Question 6 is 15 marks)

7.

Diagrams not drawn to scale

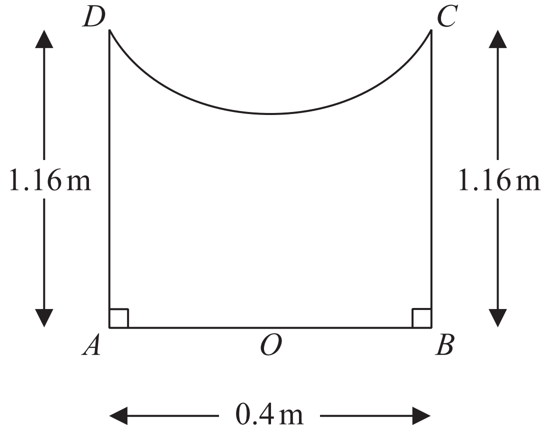


Figure 1

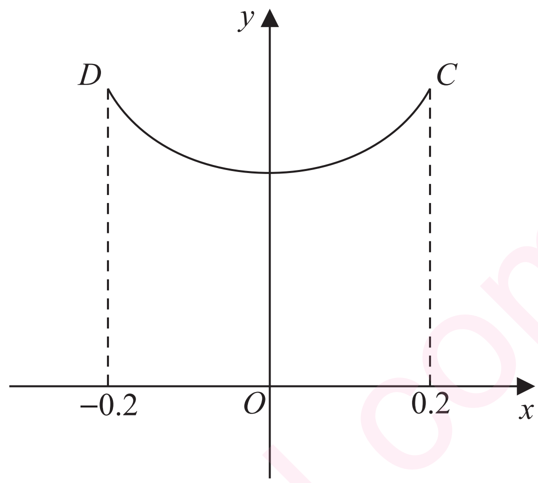


Figure 2

Figure 1 shows the central cross-section *AOBCD* of a circular bird bath, which is made of concrete. Measurements of the height and diameter of the bird bath, and the depth of the bowl of the bird bath have been taken in order to estimate the amount of concrete that was required to make this bird bath.

Using these measurements, the cross-sectional curve *CD*, shown in Figure 2, is modelled as a curve with equation

$$y = 1 + kx^2 \qquad -0.2 \leqslant x \leqslant 0.2$$

where k is a constant and where O is the fixed origin.

The height of the bird bath measured 1.16 m and the diameter, *AB*, of the base of the bird bath measured 0.40 m, as shown in Figure 1.

(a) Suggest the maximum depth of the bird bath.

(1)

(b) Find the value of k.

(2)

(c) Hence find the volume of concrete that was required to make the bird bath according to this model. Give your answer, in m^3, correct to 3 significant figures.

(7)

(d) State a limitation of the model.

(1)

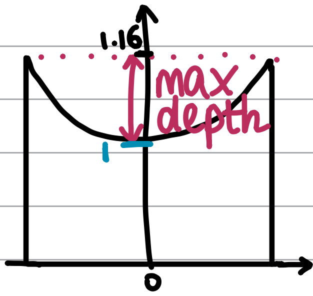
It was later discovered that the volume of concrete used to make the bird bath was $0.127\,\text{m}^3$ correct to 3 significant figures.

(e) Using this information and the answer to part (c), evaluate the model, explaining your reasoning.

(1)

Question 7 continued

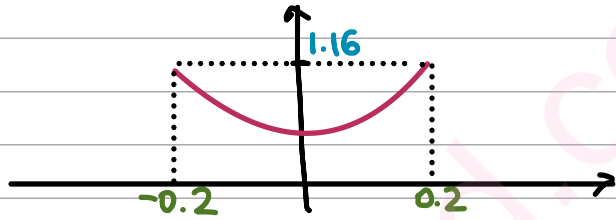
7.a)



y co-ordinate at origin

↳ $y_0 = 1 + k(0)^2$ $\quad y_0 = 1$

$\therefore$ max depth $= 1.16 - 1 = 0.16\,m$



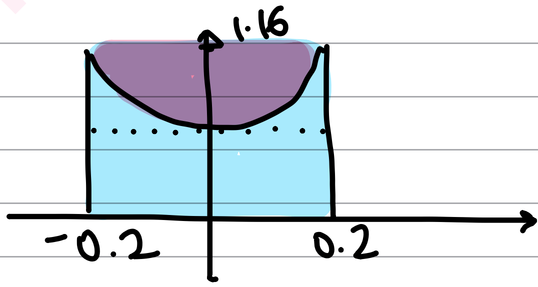
b) $y = 1 + kx^2$

$1.16 = 1 + k(0.2)^2$ OR $1.16 = 1 + k(-0.2)^2$

$0.04k = 0.16$ $\qquad$ $0.04k = 0.16$

$k = 4$ $\qquad$ $k = 4$

c) $\boxed{V_{\text{about } y\text{-axis}} = \pi \int_a^b x^2 \, dy}$



$y = 1 + 4x^2$

$x = \sqrt{\dfrac{y-1}{4}}$

$V = \pi \int_1^{1.16} \left(\sqrt{\dfrac{y-1}{4}}\right)^2 dy$

$V = \dfrac{\pi}{4} \int_1^{1.16} y - 1 \; dy$

$V = \dfrac{\pi}{4} \left[\dfrac{y^2}{2} - y \right]_1^{1.16} = \dfrac{\pi}{4}(0.0128)$

Question 7 continued

$= 0.0032\pi$

V = cylinder → radius = 0.2
height = 1.16

$\therefore V = \pi(0.2)^2 \times 1.16$
$= 0.0464\pi$

Volume of bird bath = $\text{Vol}_{\text{cylinder}} - \text{Vol}_{\text{rev.}}$

$= 0.0464\pi - 0.0032\pi$

$= 0.0432\pi \text{ m}^3$

$= 0.136 \text{ m}^3$

d) The surface of the bowl may not be smooth & measurements may not be accurate

e) $\% \text{ diff.} = \frac{|\text{calculated} - \text{actual}|}{\text{actual}} \times 100\%$

$\%d = \frac{0.136 - 0.127}{0.127} \times 100\% \approx 7\%$

$\therefore$ not a good estimate because the volume needed is 7% lower than predicted by the model

(Total for Question 7 is 12 marks)

8. (a) Shade on an Argand diagram the set of points

$$\left\{z \in \mathbb{C} : |z - 4\mathrm{i}| \leqslant 3\right\} \cap \left\{z \in \mathbb{C} : -\frac{\pi}{2} < \arg(z + 3 - 4\mathrm{i}) \leqslant \frac{\pi}{4}\right\}$$

(6)

The complex number w satisfies

$$|w - 4\mathrm{i}| = 3$$

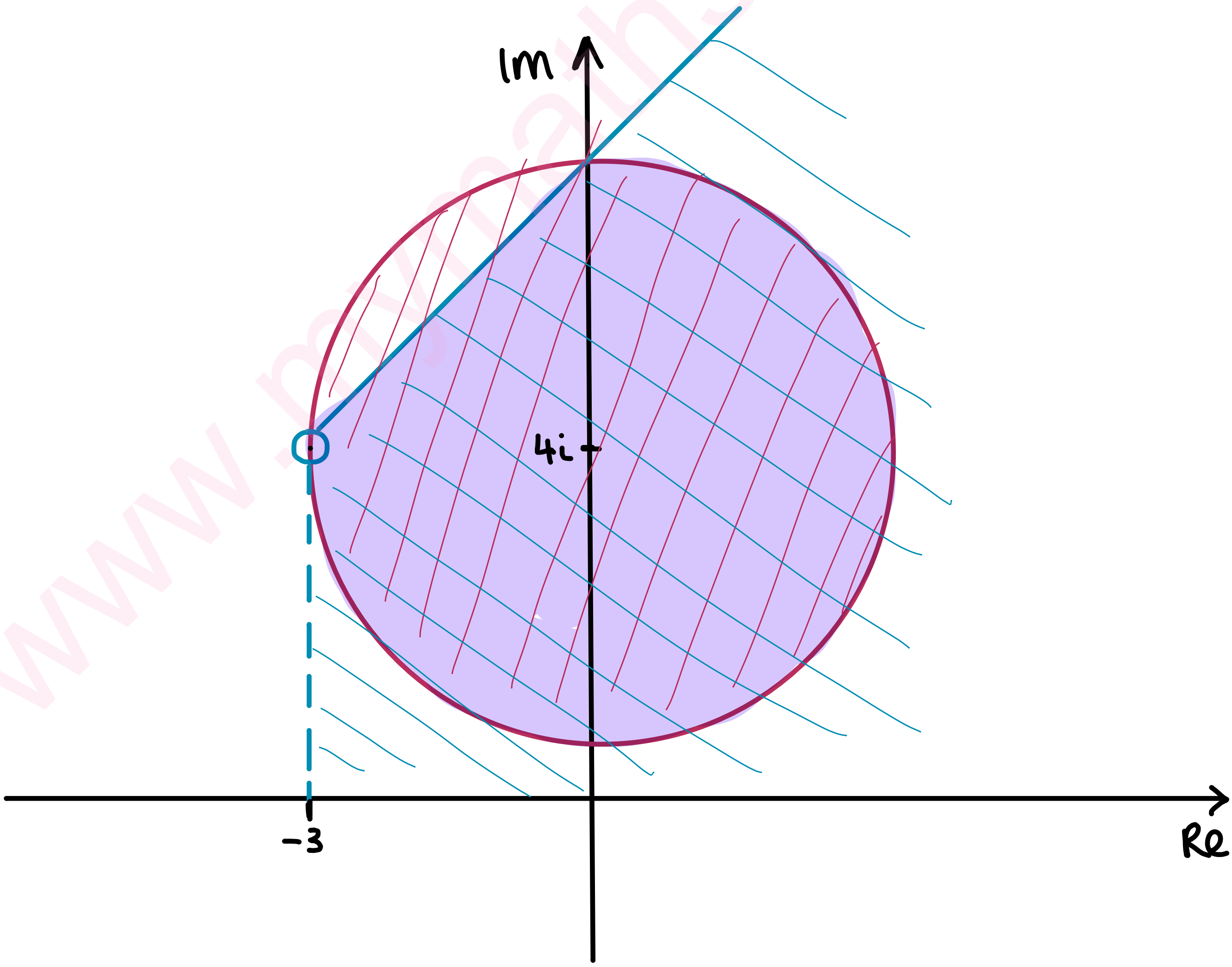
(b) Find the maximum value of $\arg w$ in the interval $(-\pi, \pi]$.
Give your answer in radians correct to 2 decimal places.

(2)

AND

8.a) $\{z : |z-(4i)| \leq 3\} \cap \{z : -\frac{\pi}{2} < \arg(z-(-3+4i)) \leq \frac{\pi}{4}\}$

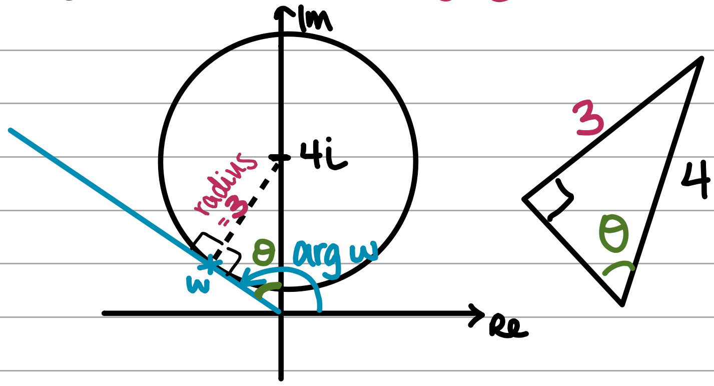
circle → centre $(0,4)$
radius ≤ 3
∴ inside circle



Question 8 continued

b) $|w-4i| = 3$

$\therefore$ w lies on the circle



$\sin\theta = \frac{opp.}{hyp.} = \frac{3}{4}$

$\theta = \sin^{-1}\left(\frac{3}{4}\right)$

$\theta = 0.848$ rad

$\arg w = \frac{\pi}{2} + 0.848$

max $\arg w = 2.42$ rad.

(Total for Question 8 is 8 marks)

9. An octopus is able to catch any fish that swim within a distance of 2 m from the octopus's position.

A fish F swims from a point A to a point B.

The octopus is modelled as a fixed particle at the origin O.

Fish F is modelled as a particle moving in a straight line from A to B.

Relative to O, the coordinates of A are (–3, 1, –7) and the coordinates of B are (9, 4, 11), where the unit of distance is metres.

(a) Use the model to determine whether or not the octopus is able to catch fish F. **(7)**

(b) Criticise the model in relation to fish F. **(1)**

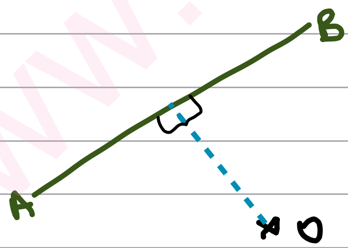
(c) Criticise the model in relation to the octopus. **(1)**

9.a) Path of fish, F is from A to B

$$\vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$$

$\therefore$ fish, F travels along line : $r = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$

$$\therefore r = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$$



Shortest distance OF is when OF is perpendicular to AB

$$OF = \begin{pmatrix} -3+4\lambda \\ 1+\lambda \\ -7+6\lambda \end{pmatrix} \qquad AB = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$$

$$OF \cdot AB = \begin{pmatrix} -3+4\lambda \\ 1+\lambda \\ -7+6\lambda \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix} = 4(-3+4\lambda) + (1+\lambda) + 6(-7+6\lambda)$$

$$= -53 + 53\lambda = 0$$

$$\lambda = 1$$

Question 9 continued

$\therefore\ OF = \begin{pmatrix} -3+4(1) \\ 1+(1) \\ -7+6(1) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

min distance $= \left| \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \right|$

$= \sqrt{1^2 + 2^2 + (-1)^2}$

$= \sqrt{6} = 2.449\ldots > 2$

$\therefore$ octapus cannot catch the fish F

b) the fish will probably not swim in a perfectly straight line from A to B

c) the octapus is not a fixed particle, and it may move

(Total for Question 9 is 9 marks)

TOTAL FOR PAPER IS 80 MARKS